数理解析研究所講究録
第 1599 巻 2008 年 176-181

多層型矩形分割に対する 16 分格子グラフ表現

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Abstract

Multilayer rectangular dissections have been considered such as models of multi page books in spreadsheets, 3D facility layout and stratum maps in terrain surfaces.

In this paper, we deal with a representation of the multilayer rectangular dissections that are effective for ruled line oriented operations and introduce hexadecimal grids corresponding to multilayer rectangular dissections. And we show an cell unification algorithm that runs in O(1) time, and also show a layer deletion algorithm. Then, we also propose a processing system. Furthermore, we propose 24-ary Grid Graph corresponding to rectangular solid dissection.

1. Introduction

Multilayer rectangular dissection is an effective model to represent a multi page book in spreadsheets, facility layout and stratum maps. The transformation operation over sheets, stratum maps and 3D facility layout of buildings often preserve ruled lines. Our work is placed in raster graph. In [1, 2], it was shown that Octgrids for the heterogeneous rectangular dissections are effective for ruled line oriented transformations. Another related works are quad tree, and rectangular duals graphs.

In this paper, we extend the Octgrids to 3-dimension in the sense of multi layers. We propose Hexadecimal grid graph representation of the multilayer rectangular dissections that is effective for ruled line oriented operations [9]. And we extend Octgrids to solid models.

In Section 2, we review rectangular dissections and Octgrids as preliminaries. Section 3 contains several definitions for Hexadeci-grids. We propose in Section 4 two algorithms cell unification algorithm and layer deletion algorithm with a data structure called H8CODE corresponding Hexadeci-grids. In Section 5, we explain application system with H8CODE. And Section 6, we propose 24-ary grid graph that correspond a rectangular solid dissection. Hexadeci-grids can't represent a solid model, so we construct 24-ary grid graph and represent solid terrain surfaces. Finally, we sum up main points in this paper and future works.

2. Preliminaries

We provide this Section to explain octal grid graphs for the rectangular dissections.

2.1 Rectangular Dissection

In this paper, we deal with tables with heterogeneous rectangular dissections (compare Fig. 3 with 4). Here, we review the definitions of tables and tabular diagrams given by Motohashi- Tsuchida-Yaku [1, 2].

Definition 2.1.1

An (n,m) − *table* is a $\{(i, j)|1 \leq j \leq n,\ 1 \leq j \leq m\}$ of integer pairs. A table is an (n, m) − table for some n and m. A *partial table* is a subset S of an (n, m)-table, where S is in the form of $\{(i, j)|\ k \leq i \leq l,\ s \leq j \leq t\}$ for integers $1 \leq k \leq l \leq n$, $1 \leq s \leq t \leq m$. A *partition P* over a table T is a pair wise disjoint collection$S_1, S_2, \ldots, S_N$ of partial tables, where $S_1 \cup S_2 \cup \ldots \cup S_N$, and each S_i is called a *cell*. We call n the row number and m the column number.

Definition 2.1.2

The *row ruled line* of an (n, m) - table T is a map l_{row} : 0, 1, . . . $n \rightarrow R$ such that $l_{row}(i) \leq l_{row}(i+1)$ for $0 \leq i \leq n - i$. The *column ruled line* is a map $l_{column}(j)$: 0, 1, . . . ,$m \rightarrow R$ such that $l_{column}(j) \leq l_{column}(j + 1)$ for $0 \leq j \leq m - 1$. A *ruled line* is pair $l = (l_{row}, l_{column})$. A tabular diagram is a triple $D = (T, P, l)$ of a table T, a partition P over T, and a grid g of T.

Terminology 2.1.3

Let c be a cell $c = (i, j)| k \leq i \leq l, s \leq j \leq t$. The north wall of c, $nw(c)$, denote $l_{row}(k - 1)$. The south wall of c, $sw(c)$, denote $l_{row}(l)$. The east wall of c, $ew(c)$, denote $l_{column}(t)$. The west wall of c, $ww(c)$, denote $l_{column}(s - 1)$.

2.2 Octal Grid Graph

Let us also review the definition of attribute graphs representing tabular diagrams.

Definition 2.2.1

Let $D = (T, P, l)$ is A rectangular dissection. A Octgrid $G = (V_D, L, E_D, A_D, a_D)$ for D is a multi edge undirected grid graph, where V_D is identified by a partition P (We denote a node corresponding to a cell c in P by vc), $L = \{enw, esw, eew, eww\}$, E_D $E_D \subseteq V_D \times L \times V_D$ is a set of undirected labeled edges of V_D of the form $[vc, l, vd]$, where vc and vd are in V_D and l is in L. E_D is defined by the following Rules 1-4, $A_D = R^4$ and $\alpha_D : V_D \rightarrow R^4$ are defined for $vc \subseteq V_D$ by $\alpha_D = (nw(c), sw(c), ew(c), ww(c))$, where vc is only perimeter cells.

Rule 1

If $nw(c) = nw(d)$, that is, c and d have the equal north wall, and there is no cell between c and d which have the equal north wall, then $[vc, enw, vd]$ is in E_D and $\lambda_D = enw$. In this case $[vc, enw, vd]$ is called a *north wall edge*.

Rule 2

If $sw(c) = sw(d)$, that is, c and d have the equal south wall, and there is no cell between c and d which have the equal north wall, then [vc, esw, vd] is in E_D and $\lambda_D = esw$. In this case [vc, esw, vd] is called a *south wall edge*.

Rule 3

If $ew(c) = ew(d)$, that is, c and d have the equal east wall, and there is no cell between c and d which have the equal north wall, then [vc, eew, vd] is in E_D and $\lambda_D = enw$. In this case [vc, eew, vd] is called a east wall edge.

Rule 4

If $ww(c) = ww(d)$, that is, c and d have the equal west wall, and there is no cell between c and d which have the equal north wall, then [vc, eww, vd] is in E_D and $\lambda_D = enw$. In this case [vc, eww, vd] is called a *west wall edge*.

The degree of edges is at most eight in Octgrid graph. Figure 1 shows an example of rectangle division and the corresponding Octgrids. Figure 1 shows an example of rectangular dissection and the corresponding Octgrids.

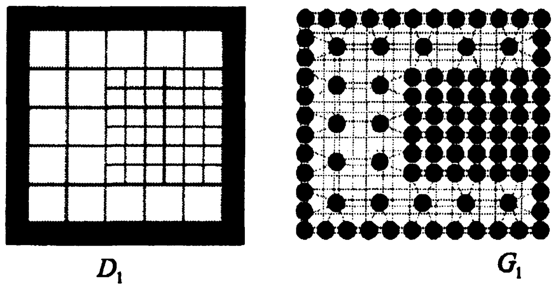

Figure 1. Rectangular dissection D_1 and octgrid G_1 that corresponds to D_1.

3. Hexadeci-Grids

In this section, we provide *Hexadeci-Grids*. First, we explain *multilayer rectangular dissection*. Multilayer rectangular dissection is an extension of rectangular dissection. We give rectangular dissections hierarchical structure and arrange it in multi-layer type. We define multilayer rectangular dissections as follows.

Definition3.1.1

An *l-layer(n , m)-table* is a set $\{i , j, k \mid 1 \leqq i \leqq n, 1 \leqq j \leqq m , 1 \leqq k \leqq l\}$ of integer trilogy. A *multi layer table* is an l-layer (n, m)-table for some l, m, n. An *k-th layer partial table of l-layer (n, m)-table* is a subset S of an *(n, m)-table* where S is in the form of $\{(i, j)| u \leq i \leq v, s \leq j \leq t\}$ for integers $1 \leq u \leq v \leq n, 1 \leq s \leq t \leq m$. A partition P over a *l-layer (n, m)-table* T is a pair wise disjoint collection $S_1, S_2, ..., S_N$ of k-th layer partial T, where $1 \leqq i \leqq n$, and $S_1 \cup S_2 \cup \ldots \cup S_N = T$. We call each S_i *cell.*

Definition3.1.2

The *row ruled lines* (n, m)*-table* T is a map $l_{row}(i) : \{0, 1, \ldots, n\} \rightarrow R$ such that $l_{row}(i) \leqq l_{row}(i+1)$ for $0 \leqq i \leqq n-1$.

The *column ruled lines* (n, m)*-table* T is a map $l_{column}(j) : \{0, 1, \ldots, m\} \rightarrow R$ such that $l_{column}(j) \leqq l_{column}(j+1)$ for $0 \leqq j \leqq m-1$.

A *ruled lines* is a pair $l = (l_{row}, i_{column})$. A *tabulardaigram* is a tuple $E = (T, P, l)$ of a table T, a partition T and a ruled line l of T.

Definition3.1.3

Let c be a cell $c = \{(i, j) \mid k \leqq i \leqq l, s \leqq j \leqq t\}$. The *northeastern corner* of c, $nec(c)$, denote the corner which intersects of $l_{row}(k-1)$ and $l_{column}(t)$. The *northwestern corner* of c, $nwc(c)$, denote the corner which intersects of $l_{row}(k-1)$ and $l_{column}(s-1)$. The *southeastern corner* of c, $sec(c)$, denote the corner which intersects of $l_{row}(l)$ and $l_{column}(t)$. The *southwestern corner* of c, $swc(c)$, denote the corner which intersects of $l_{row}(l)$ and $l_{column}(s-1)$.

Figure 4 shows an example of multilayer rectangular dissections.

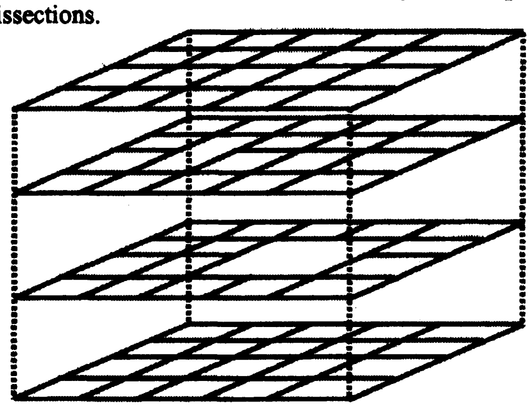

Figure 2. An example of multilayer rectangular dissections (three layers).

Next, we explain hexadecimal Grids.

Let $D = (T, P, l)$ be a k-layered multilayer rectangular dissection. A *Hexadeci-Grid* $G_D = (V_D, L, E_D, A_D, \alpha_D)$ for D is a multi-edge undirected grid graph. We explain definitions of L, E_D, A_D, and α_D. First, we define the nodes V_D. We put a node corresponding to each cell. Next, we define label L.

$L = \{$ *enw, esw, eew, eww nec, nwc, sec, swc* $\}$

Next, we define edges E_D. $E_D \subseteq V_D \times L \times V_D$ is a set of undirected labeled edges of V_D of the form $[vc, l, vd]$, where vc and vd are in V_D and l is in L.

(i) We define Edges in E_D between nearest cells in D that have corner horizontally in common similarly Octgrids

(ii) We define edges in E_D between nearest cells in D that have corners vertically in common.

Rule 5

It is assumed that Cell c and d are located in the different layer. If $nec(c) = nec(d)$ and there is no cell between c and d which have the equal x - y coordinate of a northeastern, then $[vc, nec, vd]$ is in E_D. In this case, $[vc, nec, vd]$ is called *northeastern corner edge.*

Rule 6

It is assumed that Cell c and d are located in the different layer. If $nwc(c) = nwc(d)$ and there is no cell between c and d which have the equal x - y coordinate of a northwestern, then $[vc, nwc, vd]$ is in E_D . In this case, $[vc, nwc, vd]$ is called *northwestern corner edge.*

Rule 7

It is assumed that Cell c and d are located in the different layer. If $sec(c) = sec(d)$ and there is no cell between c and d which have the equal x - y coordinate of a southeastern, then $[vc, sec, vd]$ is in E_D. In this case, $[vc, sec, vd]$ is called *southeastern corner edge.*

Rule 8

It is assumed that Cell c and d are located in the different layer. If $swc(c) = swc(d)$ and there is no cell between c and d which have the equal x - y coordinate of a southwestern, then $[vc, swc, vd]$ is in E_D. In this case,$[vc, swc, vd]$ is called *southwestern corner edge.*

$A_D = R^4$ and $\alpha_D : V_D \rightarrow R^4$ are defined for $vc \in V_D$ by $\alpha_D = (nw(c), sw(c), ew(c), ww(c))$.

By these rules, we decide two values in an upper course and a lower course in each corner. The number of edges that is from nodes of Hexadecimal dissection is at most 16. Figure 5 shows relations of edges that are from nodes of Hexadeci-grids and cells of multilayer rectangular dissections.

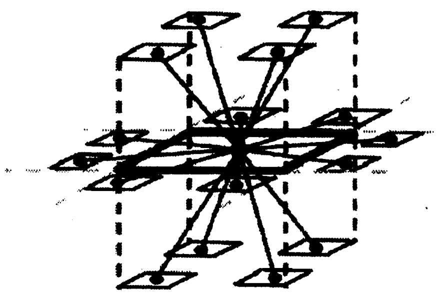

Figure 3. Link around a cell for hexadecimal grids.

4. Algorithms

This Section explains Algorithm with Hexadeci-grids. But Hexadeci-grid is an undirected grid graph, so we extend Hexadeci-grids to32-ary grid graph that is a directed grid graph. First, we define 32ary-grid graph. Next, we propose algorithms.

32-ary Directed Grid Graph

GD is represented by 32-ary directed grid graph. G_{DD} = (V_D, *L*, *Direction*, *A*, E_{DD}), where Direction = {*Forward*, Backward}.

$E_{DD} \subseteq V_D \times L_D \times A_D \times Direction \times V_D$ is defined as follows.

If [*s*, *NorthEasternCornerEdge*, *t*] is in E_D, then

(1) (*s*, *NortheasterncornerEdge*, *Forward*, *t*) is in E_{DD} ($sx \leqq tx$)

(2) (*s*, *NorthEasternCorneredge*, *Backward*, *t*) is in ED_D ($s_x \geqq t_x$)

CELL UNIFICATION ALGORITHM

In a past works, Motohashi, Tsuchida and Yaku introduced algorithm called Unify Cells, which operate unifying two cells [3, 4]. We improve UnifyCells and make CELL UNIFICATION ALGORITHM.

ALGORITHM

CELL UNIFICATION ALGORITHM

INPUT

G_D =(V_D, E_D, *L*, A_D, α_D) : a Hexadeci-grid for multilayer rectangular dissection *D*, v_x ,v_y : a node in G_D and *ww*(*x*) = *ww*(*y*), *ew*(*x*) = *ew*(*y*), and *sw*(*x*) = *nw*(*y*).

OUTPUT

G_F = (V_F, E_F, *L*, A_F, α_F) : a Hexadeci-grid for multilayer rectangular dissection *D*, where *F* is obtained from *D* by unify cells *x* and *y* into *x*.

METHOD

1. Change edges of the horizontal direction around v_x and v_y.
2. Delete edges of the horizontal direction around v_y.
3. Change edges of the vertical direction around v_x and v_y.
4. Delete edges of the vertical direction around v_y.
5. Delete node v_y.

ALGORITHM

LAYER DELETION ALGORITHM

INPUT

G_D = (V_D, E_D, *L*, A_D, α_D) : a Hexadeci-grid for multilayer rectangular dissection *D*, v_x: a node in G_D .

OUTPUT

G_F = (V_F, E_F, *L*, A_F, α_F) : a Hexadeci-grid for multilayer rectangular dissection *D*, where *F* is obtained from *D* by delete *k*-th layer .

METHOD

1. Delete edges of the horizontal direction around v_x .
2. Delete edges of the vertical direction around v_x .
3. Delete node v_x in G_D.
4. Repeat 1~3 steps for all cells in G_D

5. Processing System

In this Section, We propose *H8CODE* that is a data format corresponding Hexadeci-grids. H8CODE is based H7CODE [3] that is a data format corresponding Octgrids.

H8CODE express block structure, and consists of four blocks. The explanation of each block is as follows.

(1)**Header Block**

The header block has basic information on multilayer rectangular dissection and consists of four fields (expect spare).

(2) **List Block**

The list block has information on the attributes of cells. The number of cells equals the number of records. It has 48 fields per a record.

(3)**Content Block**

The content block has information on the texture to rapping 3D terrain surfaces.

(4)**Tabular Layer Block**

The tabular layer block has information to manage multi rectangular dissection.

Figure 6 shows detail of H8CODE and Figure 7 shows list structure for Hexadeci-grids.

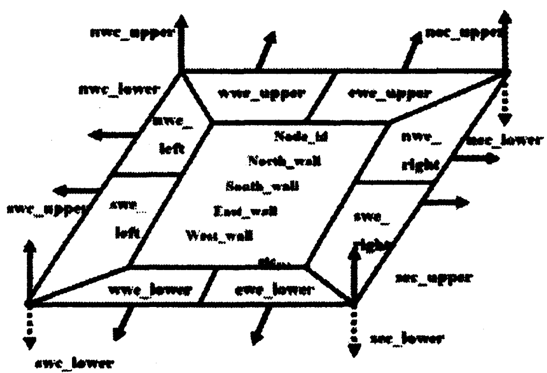

Figure 4. Fields of a record

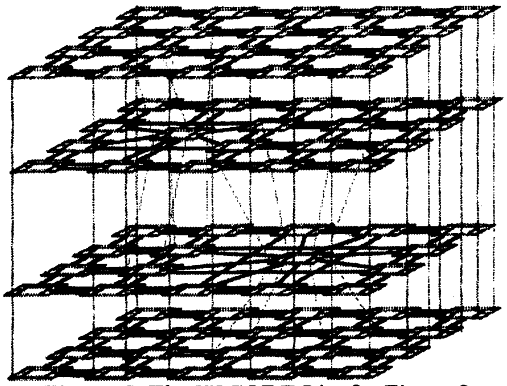

Figure 5. The H8CODE List for Figure 2.

Next, we explain an application system with *H8CODE*. Hexadeci-grids correspond to multilayer terrain surfaces. Therefore this application can express chronologically topological maps and stratum state by 3D topographical maps [4, 5, and 6].

First we express the flow charts from DEM data to a VRML file in figure 6.

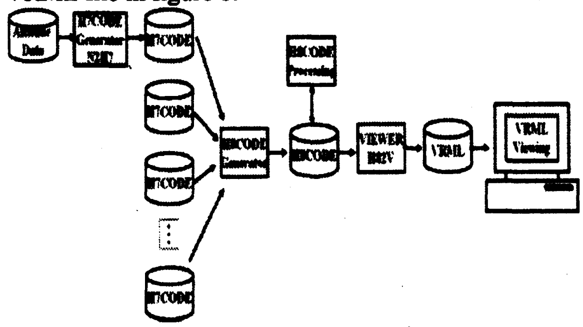

Figure 6. System over view.

Next, we show an example of terrain surfaces with H8CODE. Figure 7 is an example of multilayer terrain surfaces. The upper map is recognized valley of Mt. Fuji, and the lower map recognized ridge of Mt. Fuji[7, 8].

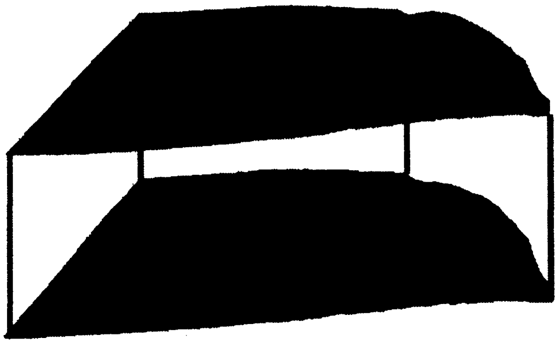

Figure 7. An example of terrain surfaces.

6. 24-ary Grid Graph

Next, we propose *24-ary grid graph*. Hexadeci-grids can't represent a solid surface. So 24-ary grid graph is an effective model to represent a solid surface.

And we purpose that 24-ary grid graph is a base of system that processes terrain surfaces by meters above the sea level data. We define 24-ary grid graph as follows.

Definition 6.1.1

A rectangular solid dissection is a collection $D = \{S_1, S_2, \ldots, S_N\}$ of mutually disjoint rectangular solids in a rectangular solid dissection, where $S_1 \cup S_2 \cup \ldots \cup S_N = D$.

Definition 6.1.2

Let $D = \{S_1, S_2, \ldots, S_N\}$ be a rectangular solid dissection. A tetraicosa-grid for D is a multi edge labeled grid graph $G_D = (V_D, L, E_D, A_D)$, defined as follows:

(1) V_D is defined as follows:

If a rectangular solid s is in D, then v_s is in V_D.

(2) L = *{equivalent upper north beam, equivalent upper south beam, equivalent upper east beam, equivalent upper west beam, equivalent north east corner, equivalent north west corner, equivalent south west corner, equivalent south east corner, equivalent lower north beam, equivalent lower south beam, equivalent lower east beam, equivalent lower west beam}*

(3) $E_D \subseteq V_D \times L \times V_D$ is a set of undirected labeled edge, defined as follows : If s and t are nearest solids in D such that *s* and *t* have upper north beam in common, then [*s*, *equivalent upper north beam*, *t*] is in E_D.

Figure 10 shows link around a cell for 24-ary Grid Graph.

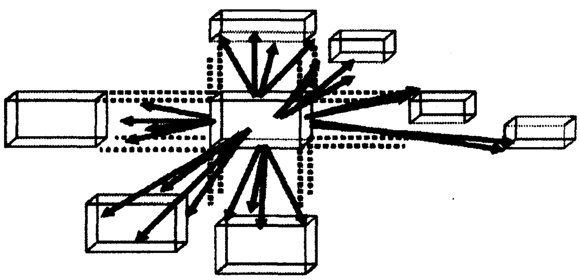

Figure 8. Link around a cell for24-ary Grid Graph.

48-ary directed grid representation

G_D is represented by 48-ary directed grid graph G_{DD} = (V_D, L, Direction, A_D, E_{DD}), where Direction = {Forward, Backward}.

$E_{DD} \subseteq V_D \times L \times A_D \times$ Direction $\times V_D$ is defined as follows

If [s, EquivalentUpperNorthBeam, t] is in E_D, then

(1) (s, EquivalentUpperNorthBeam, Forward, t) is in E_{DD} (sx $\leq$ tx)

(2) (s, EquivalentUpperNorthBeam, Backward, t) is in E_{DD} (sx $\geq$ tx)

7. Conclusion

We proposed hexadeci-grids for Multilayer dissection and a data format called corresponding to hexadecimal grids. And we express algorithm Delete layer and algorithm unify cells. And we proposed tetraicosa-grids corresponding to rectangular solid dissection.

As for future works, we implement the system of multiresolution terrain surfaces for stratum maps and chronologically topographical maps. And we will construct insert layer algorithm.

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